

MATHEMAGIC ACTIVITY BOOK

CLASS - X

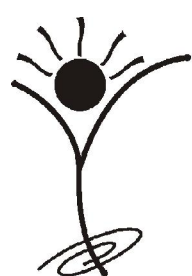
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e.mail: info@eduhealfoundation.org, website: www.eduhealfoundation.org



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10

CONTENTS

Topics	Page No.
■ Syllabus Guidelines	04
■ Maths is All Around Us	06
■ Top Mathematical Mistakes	10
■ Can You Outperform The Japanese?	12
■ How Lucky are You	13
■ Pythagorean Theorem Puzzle	14
■ Mathemagics	15
■ Mathematics Challenge Problems	17
■ Maths is Joke	19
■ Mathematicians of India	20
■ Brain Twisters	22
■ Recognise The Pattern	23
■ Interactive Activities : Maths Puzzle	24
■ Chances of Coincidence	25
■ 36 Methods of Mathematical Proof	26
■ The Mathematical Magic of The Fibonacci Numbers	27
■ Answers	39
■ Sample Questions	31

3

SYLLABUS GUIDELINES

CLASS - X

Based on CBSE, ICSE & GCSE Syllabus

UNIT I: NUMBER SYSTEMS

1. REAL NUMBERS

Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples.

Irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$ decimal expansions of rational numbers in terms of terminating/non-terminating recurring decimals.

UNIT II: ALGEBRA

3. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of a polynomial with particular reference to quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

4. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables. Geometric representation of different possibilities of solutions/inconsistency.

Algebraic conditions for number of solutions. Solution of pair of linear equations in two variables algebraically – by substitution, by elimination and by cross multiplication. Simple situational problems must be included. Simple problems on equations reducible to linear equations may be included.

5. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$. Solution of the quadratic equations (only real roots) by factorization and by completing the square, i.e. by using quadratic formula. Relationship between discriminant and nature of roots. Problems related to day to day activities to be incorporated.

6. ARITHMETIC PROGRESSIONS

Motivation for studying AP. Derivation of standard results of finding the n th term and sum of first n terms.

UNIT III: TRIGONOMETRY

1. TRIGONOMETRIC RATIOS

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0° & 90° . Values (with proofs) of the trigonometric ratios of 30° , 45° & 60° . Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$ Only simple identities to be given. Trigonometric ratios of complementary angles.

3. HEIGHTS AND DISTANCES

Simple and believable problems on heights and distances.

Problems should not involve more than two right triangle. Angles of elevation/depression should be only 30° , 45° , 60° .

UNIT IV: COORDINATE GEOMETRY

1. LINES (In two-dimensions)

Review the concepts of coordinate geometry done earlier including graphs of linear equations. Awareness of geometrical representation of quadratic polynomials. Distance between two points and section formula (internal). Area of a triangle.

10

UNIT V: GEOMETRY

1. TRIANGLES

Definitions, examples, counterexamples of similar triangles.

1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then other two sides are divided in the same ratio.

2. If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

3. If in two triangles, then the corresponding angles are equal, their corresponding sides are proportional then the triangles are similar.

4. If the corresponding sides of two triangles are proportional, their corresponding angles are equal then the two triangles are similar.

5. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.

6. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

7. The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.

8. In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

9. In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite two the first side is a right triangle.

2. CIRCLES

Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.

1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

2. The lengths of tangents drawn from an external point to circle are equal.

3. CONSTRUCTIONS

1. Division of a line segment in a given ratio (internally)

2. Tangent to a circle from a point outside it.

3. Construction of a triangle similar to a given triangle.

UNIT VI: MENSURATION

1. AREAS OF PLANE FIGURES

Motivate the area of a circle; area of sectors and segments of a circle.

Problems based on areas and perimeter/circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° & 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken.)

2. SURFACE AREAS AND VOLUMES

(i) Problems on finding surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone.

(ii) Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken.)

UNIT VII: STATISTICS & PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

2. PROBABILITY

Classical definition of probability. Connection with probability as given in Class IX. Simple problems on single events, not using set notation.

5

Maths is All Around Us

Maths is everywhere. Without realising, we use maths every day, and it plays a part in nearly all our daily activities. Everytime we pick up the phone, use the internet, manage money, decide to take a risk, check the weather report, go to the doctors or travel anywhere, maths plays its part

The weather

You wake up in the morning and get ready to go out. You check the weather report to decide what to wear and what to take - an umbrella? Sunglasses? A hat? The forecasts you read in the paper are a result of solving complicated equations involving the way air, clouds and water move around the planet - part of an area of mathematics called fluid dynamics.

Long-term weather trends can affect our whole environment. For instance, climate change might lead to more storms and floods. These problems are part of the work of environmental statisticians who study data received from around the world and try to predict what might happen over the next several years.

Buildings and construction

If it is rainy and cold outside, you will be happy to stay at home a while longer and have a nice hot cup of tea. But someone has built the house you are in, made sure it keeps the cold out and the warmth in, and provided you with running water for the tea. This someone is most likely an engineer. Engineers are responsible for just about everything we take for granted in the world around us, from tall buildings, tunnels and football stadiums, to kettles and access to clean drinking water. They also design and build vehicles, aircraft, boats and ships. What's more, engineers help to develop things which are important for the future, such as generating energy from the sun, wind or waves. Maths is involved in everything an engineer does, whether it is working out how much concrete is needed to build a bridge, or determining the amount of solar energy necessary to power a car.

Money

Once out of the door you walk past a news stand where the headlines “house prices boom again” screams from the front pages. House prices are determined by supply and demand, and the one thing anyone buying a house needs is money. Most people get a loan from a bank to buy a house, and then have to pay back the money they borrowed plus some interest and fees. To choose which bank to borrow from, and to decide if you can afford to borrow at all, you need to understand compound interest.

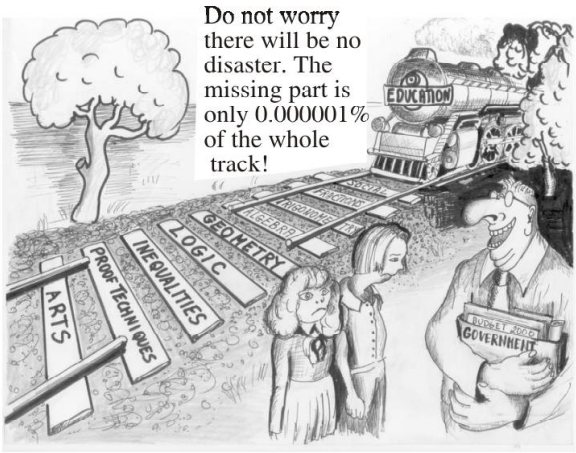
The housing market is part of the bigger economic picture. How the economy is doing affects how much things cost, how much we are paid and how much the government spends - and maths is used to monitor the economy and predict how it will change. A large part of the world’s economy is invested in the stock market, and highly skilled mathematicians are employed to try to understand, and even predict, movements in the stock market.

Planning

Whether you are managing money (trying to save for a holiday), resources (trying to make those rations last) or time (deciding how much time to spend on studying for each subject), you are doing calculations, sometimes automatically in your head, trying to optimise your outputs (money saved, grades achieved) given your inputs (pay from part-time job, hours in the day). In industry this is called operational research, and is used to improve the processes used in manufacturing, in how businesses are run, and in making the best use of resources such as beds in a hospital or police on the beat.

Transport

You wait for the bus. Maths is used to timetable buses, trains and planes, taking into account the limited stocks of vehicles, the cost of fuel and the number of people wanting to use them for



various journeys across the day. It is used to decide the timing of traffic lights, and to understand traffic flow and congestion. This is another application of operational research.

Maths also affects the designs of these vehicles, particularly planes, cars and boats. Aerodynamics, another application of fluid dynamics, is obviously vital in designing a plane that can actually fly, but is also important in designing any sort of vehicle to move as smoothly through the air as possible so that it can go faster and take less fuel.

Understanding chance and risk

You get off the bus and someone is selling tickets in a raffle for a new car. Should you buy one? What are the chances of you winning in this, or in any other, lottery? All games of chance are governed by probabilities of events happening. If you understand probability, you have a much better chance of coming out on top, or in deciding that it's not worth spending your money at all.

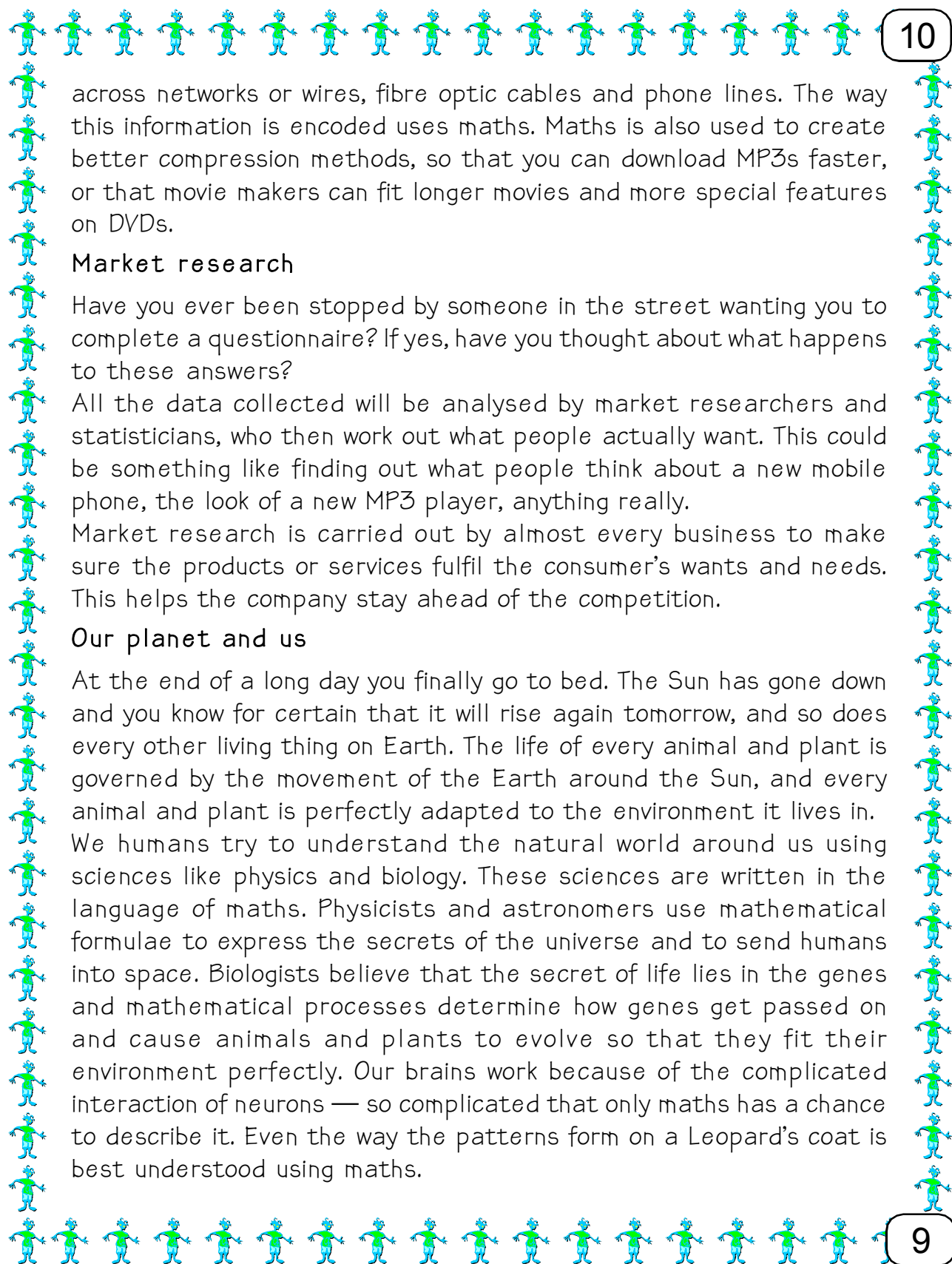
The bus got stuck in traffic, and you arrived at your destination really late. Should you ride your bike next time, you wonder. But is that too risky? And, if you do, should you wear a helmet?

Your friend calls you to say she has had some negative test results from the doctor. Is she definitely sick, or what are the chances that the test is wrong and she is not sick at all? There are a number of treatment options available, but all have different rates of success and different side effects. To decide which one to take, she will need to weigh up the risks of side effects of each treatment against the chances of success, using the probabilities of each of these.

Business and industry also need to make decisions about chance and risk. They need mathematicians - or operational researchers - to help them with this.

Communicating

Finally you get home and get a chance to check your email. Computers store all information as binary numbers, and use mathematical operations to manipulate this information, whether you are editing an email, resizing a photo or setting preferences in your web browser. Computers communicate digitally, sending packets of information



across networks or wires, fibre optic cables and phone lines. The way this information is encoded uses maths. Maths is also used to create better compression methods, so that you can download MP3s faster, or that movie makers can fit longer movies and more special features on DVDs.

Market research

Have you ever been stopped by someone in the street wanting you to complete a questionnaire? If yes, have you thought about what happens to these answers?

All the data collected will be analysed by market researchers and statisticians, who then work out what people actually want. This could be something like finding out what people think about a new mobile phone, the look of a new MP3 player, anything really.

Market research is carried out by almost every business to make sure the products or services fulfil the consumer's wants and needs. This helps the company stay ahead of the competition.

Our planet and us

At the end of a long day you finally go to bed. The Sun has gone down and you know for certain that it will rise again tomorrow, and so does every other living thing on Earth. The life of every animal and plant is governed by the movement of the Earth around the Sun, and every animal and plant is perfectly adapted to the environment it lives in. We humans try to understand the natural world around us using sciences like physics and biology. These sciences are written in the language of maths. Physicists and astronomers use mathematical formulae to express the secrets of the universe and to send humans into space. Biologists believe that the secret of life lies in the genes and mathematical processes determine how genes get passed on and cause animals and plants to evolve so that they fit their environment perfectly. Our brains work because of the complicated interaction of neurons — so complicated that only maths has a chance to describe it. Even the way the patterns form on a Leopard's coat is best understood using maths.



Top Mathematical Mistakes

Find out where you can go wrong.

- ①. Simple arithmetic errors, and calculator entry errors.

How to avoid:

- Have a rough idea of the outcome. For example, know that $\frac{3}{4}$ of $\frac{1}{2}$ is going to be less than a half and more than a quarter.
- Double check numbers if they become too difficult to do in your head.
- Do your calculations directly in the test paper near the question where there is space. This saves time and reduces rewriting errors.

- ②. Giving the answer for X when the question asks for $X/2$ or $X-Y$.

How to avoid:

When reading the last sentence of the question, underline or circle what the question asks for, then check after your calculations. After doing intense calculations to find X , it is easy to forget that the question was asking for $X - Y$.

- ③. Trying to find the value of a variable when there are more variables than equations.

How to avoid:

Whenever you see a problem with more variables than equations, do not try to solve for any one of the variables. For example, $X + Y = 10$ cannot be solved for one value of X or one value of Y . The question will ask for some combination of X and Y , such as what is $2X + 2Y$ (To solve, multiply $X + Y = 10$ by 2).

- ④. Wasting minutes doing algebra, when you could have plugged in simple numbers such as 0, 1 or 2.

Complex algebra increases the chance of making a stupid mistake, and wastes your limited time.

How to avoid:

When you see a question asking what happens when a variable doubles, make the variable 1 and then 2 instead of X and $2X$.

- ⑤. Becoming confused when there are several variables and no numbers.

Example: Ram earns X rupees for every Y ditches he digs. If Ram digs Z ditches today, how much money will he earn today?

How to avoid:

Think of and solve exactly the same way you would if the variables were numbers.

⑥. Missing the exception in multiple-choice, must-be-true questions and the hard quantitative comparison questions.

Example: If X is greater than or equal to 1, X cubed must be greater than X squared?

In the quantitative comparison section, this same question would have X^3 in one column and X^2 in the other column.

How to avoid:

Look for numbers that would change the results numbers that act differently in the functions being compared. In this case, one squared, equals one, which is the same as one cubed.

⑦. Assuming geometric relationships, such as right triangles or parallel lines, from the look of a figure.

How to avoid:

Never assume anything from the figure alone, especially when you see “figure not drawn to scale.” Right triangles are only right

triangles if identified as such or proven as such. Lines are parallel only if identified as such or proven as such.

Exception: if you cannot solve the problem, you may want to guess by making a few assumptions.

⑧. Not using the easy way to solve problems.

There is often a hard way to solve problems that takes 1 - 3 minutes of calculations and an easy way that takes just a few seconds.

How to avoid:

Practice problems and learn how to do them fast. Know there is often a simple solution and spend a second looking for it.

⑨. Reading things into questions.

Example: Is the sum of a sequence of three numbers beginning with 5 equal to 18?

If the numbers are 5, 6 and 7 the sum is 18. But the question did not say consecutive integers. This is often a trap on the medium-hard questions.

How to avoid :

Do not assume numbers are positive, integers or anything. Do not assume anything.

These tips should help you avoid a few errors. But you need to review your math and practice greatly to increase your test score.